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Non-linear, unsteady free convection in a vertical cylinder submitted to a horizontal thermal gradient: measurements in water between 6 and 21 °C and a theoretical model of convection

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Abstract—The non-linear, unsteady behaviour of water contained in a vertical cylinder of yellow brass when submitted to a horizontal initial thermal gradient is investigated by following the temperature decay in the centre of a cylinder. Experimental results are interpreted by means of a theoretical model which allows the deduction of equations for temperature, velocity, pressure and density in the nucleus. The new equations are compared with those of conduction to provide an evaluation of the convective contribution to heat transfer. Our data indicate that when a characteristic dimensionless group which has the form of a Rayleigh number reaches a critical value of 1600 ± 50, the heat transfer may be described by a pure conduction equation.

1. INTRODUCTION

CONVECTION has been studied since the last century because of its applicative importance, but also to understand the involved physical principles. We quote here the interesting work of Marangoni [1, 2], appeared between 1871 and 1878, in which convective instabilities due to combined effects of viscosity and surface energy are analysed. These effects have recently received attention by Scriven and Sternling and quoted as 'Marangoni effects' [3].

Between 1890 and 1910 Bénard studied the formation of structures generated by steady-state convection in layers, the already famous 'Bénard cells' [4]. The interpretation of these phenomena has been rather satisfactorily given only quite recently when attention is confined to the linear stability region [5, 6].

The quantitative study of the conditions for the onset of steady-state convection is based upon the solution of the Navier–Stokes equation with boundary and initial conditions which are particularly simple in the case of horizontal thin layers of a liquid subjected to vertical gradients of temperature or chemical potential [7–24].

Very recent studies on thick water layers in vertical temperature gradients, seem to indicate non-linear properties of the convective structures giving rise to conduction–convection loops in the 4° range where water exhibits a maximum density [25, 26].

The next step toward the understanding of convective phenomena can be achieved by studying non-stationary effects which are also important for several applications in meteorology, geodynamics and engineering.

It is obvious that a change of the geometry with respect to thin layers is necessary because transient temperatures must be measured quite precisely and over appropriately long time intervals. We have chosen a long vertical cylinder (height = 3 diameters) because of its simple geometry and because, as we shall demonstrate, it allows us to overcome the evident difficulties due to non-linear terms in the equations by applying a quite simple model based upon measurements and calculations already available in the literature [27].

On the experimental side we note that temperature measurements as a function of time are much more precise, simpler and cheaper than velocity measurements which are practically compulsory in thin layers.

It is obvious that this system is very different from a layer. It deals with a horizontal temperature gradient which in principle always generates vorticity. However, the convective effects below a critical situation become so insignificant that we can speak of a 'quasi-conductive system'.

In this paper we describe the results of our measurements in a set of cylinders filled with water in the temperature range 6–21°C and the physical model of non-steady convection which generates a 'universal convection curve'. Comparison of this curve with the already known 'universal conduction curve' [28] gives a modified Rayleigh critical number as a theoretical physical lower limit of convection which compares well with the conventional Rayleigh number deduced from calculations in thin layers [5].

2. APPARATUS AND MEASUREMENTS

The apparatus for non-stationary convection and pure conduction measurements is very simple and is

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NOMENCLATURE

a	Mouton and De Roëck's constant, $K^*Gr^{1/3}\,Pr^{-2/3}\,\nu/[h(R-\delta)]$
c_v	constant volume heat capacity
e_{ij}	components of the strain tensor
g	constant of gravity
Gr	Grashof number, $\beta g \Delta T h^3/\nu^2$
h	level of the liquid contained in the cylinder
J_1	first-order Bessel function
K	thermal conductivity
$\mathbf{k}$	unit vector along the z axis
K^*	pure constant in a
Nu	Nusselt number
p	hydrostatic pressure
$\mathbf{P}$	stress tensor
Pr	Prandtl number, ν/χ
R	internal radius of the cylinder
Ra	Rayleigh number, $\beta g \Delta T R^3/(\nu\chi)$
Rc	critical Rayleigh number, $(x_f/x_f')^3$
t	time
t_f	time of inflection in convective curves
t_f'	time of inflection in conductive curves
T	temperature of the liquid in the nucleus
T_0	initial temperature
T_s	surface temperature
$\mathbf{u}$	velocity in the nucleus
v	scalar velocity in the nucleus
w	$\exp[\pm(1.7-0.7\,z/h)]$
w_1	$\exp(\pm 1.35)$
x_f	inflection point of the universal convection function
x_f'	inflection point of the universal conduction function
z	level in the liquid measured from top
z^*	normalized level in the liquid, z/h
z_m	mth-order zero of the zeroth Bessel function.

Greek symbols

β	thermal expansion coefficient, $[1/\rho(T_s)](\partial\rho/\partial T)$
δ	average thickness of the 'couche limite'
δ_{ij}	Kronecker delta function
ΔT	initial temperature difference, $\lvert T_s - T_0\rvert$
∇	gradient operator
$\nabla\cdot$	divergency operator
μ	coefficient of viscosity
ν	kinematic viscosity, μ/ρ
ρ	density of the liquid
χ	thermal diffusivity coefficient
ψ	energy dissipated by viscous forces.

sketched in Fig. 1.† It consists of a closed cylindrical yellow brass container with different diameters and heights (seven sizes; diameters (cm): 1.5, 1.7, 2.0, 2.2, 2.5, 3.0, 3.5 and the corresponding heights (cm): 5.2, 5.5, 6.4, 7.2, 7.5, 9.5, 11.2) in the centre of which a thin thermistor can be situated. The cylinder is filled with water at the chosen initial temperature T_0 and allowed a suitably long time to reach a complete thermal uniformity in a large water thermostat. The experiment is then performed by suddenly dipping the cylinder into a second thermostat taken at the final temperature T_s. The electrical resistance of the thermistor is measured and recorded as a function of time.

In order to distinguish convection from conduction in the same conditions, in some experiments the cylinder is previously filled with 2–3% weight of cotton which inhibits any convective flux giving rise to pure conduction curves which are satisfactorily compared with the Bessel functions derived for a purely conductive cylinder [28].

As shown in Figs. 2 and 3, convective and conductive graphs for temperature differences of 4 and 10°C and various radii have similar trends: both exhibit an inflection point, but the slopes are strongly different

† The first experiments were performed during an educational research on water and its properties, carried out for inservice teachers' training [29].

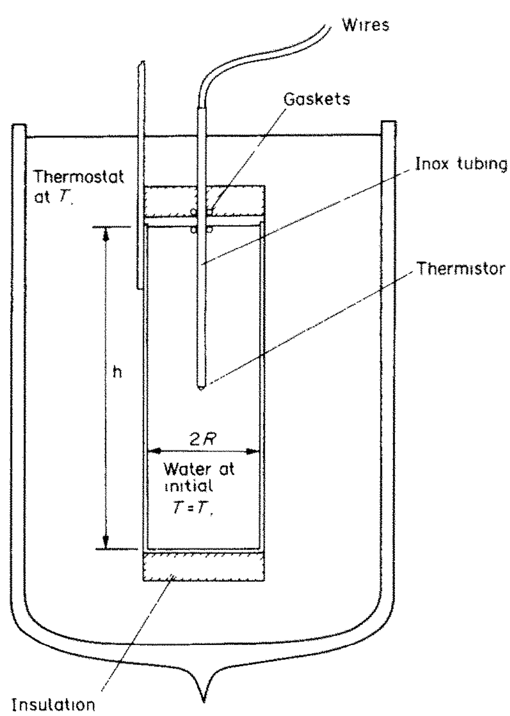

FIG. 1. Apparatus employed for conduction and convection measurements in water.

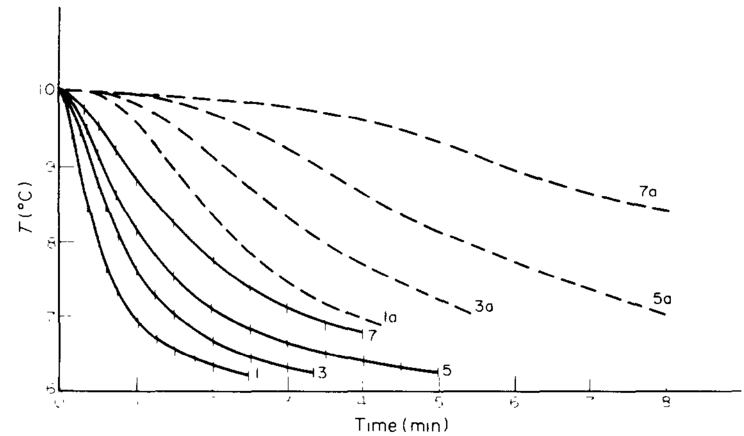

FIG. 2. Convective and conductive curves with $T_0 = 10°C$ and $T_s = 6°C$, for various diameters of cylinder. | Convection data. —— Convection curves calculated by equation (20), - - - Conduction curves calculated by equation (26): 1, 1a, 1.5 cm diameter; 3, 3a, 2.0 cm diameter; 5, 5a, 2.5 cm diameter; 7, 7a, 3.5 cm diameter. Both convection and conduction curves have a flexus.

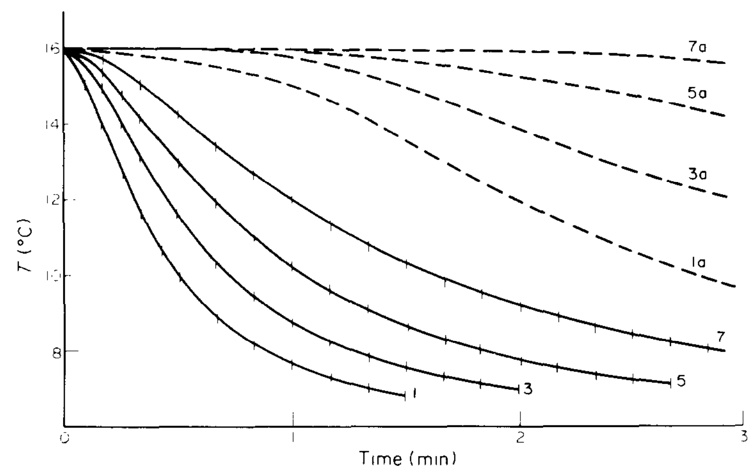

FIG. 3. Convective and conductive curves with $T_0 = 16°C$ and $T_s = 6°C$. Symbols as in Fig. 2.

and consequently the characteristic times of temperature decay are different.

Other experiments indicate that this difference is still present with a temperature difference of 1°C for cylinders whose diameter is between 2.0 and 3.5 cm, while cylinders having diameters below 2 cm become subcritical for convection (Fig. 4).

3. MODEL OF CONVECTION IN A CYLINDRICAL VESSEL†

In order to interpret the curves shown in the preceding chapter we build here a model of convection using some data and calculations from the work of Mouton and De Roëck (in the following quoted as MDR) [27]. These refer to measurements performed in a vertical cylinder submitted to large temperature gradients between 20 and 100°C. MDR describe a

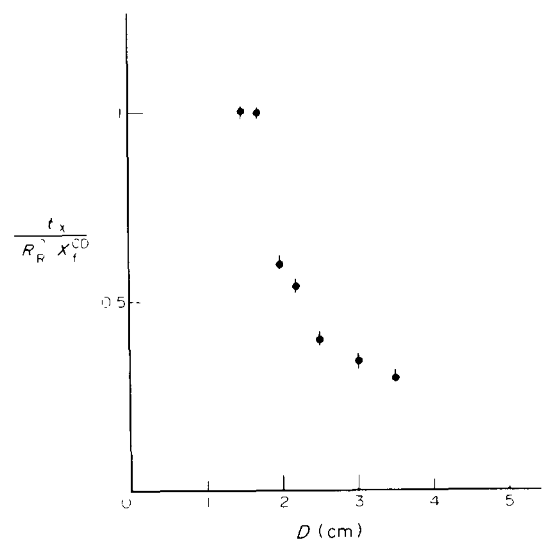

FIG. 4. Effect of diameter on convection in cylinders. The initial temperature difference was 1°C (between 7 and 6°C). Diameters below 2.1 cm give a ratio close to 1 corresponding to a quasiconductive system.

† This model has been developed by Sonnino for a graduation thesis [30].

very important series of data which serve to depict the convective flow in the cylinder as in Fig. 5 where the volume is divided into two different regions called 'couche limite', or boundary layer, and 'noyau central', or the nucleus.

If we concentrate our attention on the nucleus we see that in it the following conditions are fulfilled:

—at a given height z the temperature is constant throughout the section of the nucleus; (1)

—at a given instant t, the velocity v is constant everywhere in the nucleus. (2)

Using these two conditions and the two conservation equations:

$$\phi_E = \phi \qquad \text{(conservation of heat)}$$

$$\dot{m}_{N.C.} + \dot{m}_{C.L.} = 0 \qquad \text{(conservation of flow)}$$

where

ϕ_E = heat flux from outside to the liquid

ϕ = heat flux to increase the temperature from T_1 to T_2

$\dot{m}_{N.C.}$ = mass flow in the nucleus

$\dot{m}_{C.L.}$ = mass flow in the couche limite.

MDR obtain a solution for T in the nucleus of the cylinder as a function of time and position (modified to account for data):

$$T(z^*, t) = T_s - (T_0 - T_s)\left\{\exp(Bz^*/A) \Big/ \left[\frac{2BCR(t - t_0)v}{3h(R - \delta)^2} Gr^{1/3} Pr^{-2/3} + C\right]^3\right\} \quad (3)$$

and the velocity is:

$$v = 2A \Big/ \left[\frac{(R - \delta)^2}{\nu R} Gr^{-1/3} Pr^{2/3} + \frac{2B(t - t_0)}{3h}\right] \quad (4)$$

where A, B and C are positive constants.

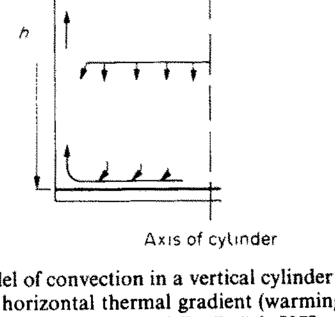

FIG. 5. Model of convection in a vertical cylinder submitted to an initial horizontal thermal gradient (warming), according to Mouton and De Roëck [27].

In the preceding chapter we have shown that our curves of convective cooling of a water cylinder (Figs. 2 and 3) exhibit an inflection point: on the other hand equation (3) has no flexus and fails to fit the data especially for small times and small initial temperature gradients.

Furthermore, the method of MDR does not give any information about density and hydrostatic pressure behaviour inside the nucleus.

We note also that in equation (3) the combination of Gr and Pr, i.e. the Nusselt number is not the one found theoretically, but it is chosen to fit the data as

$$Nu = Pr^{1/3} Gr^{1/3} B. \quad (5)$$

The theoretical calculation of Nu at large values of the radius, so that a situation similar to a plane vertical wall is approximated, gives the following result

$$Nu = B' Gr^{2/5} Pr^{7/15} (1 + 0.494 Pr^{2/3})^{-2/5}$$

where B' is a pure constant.

The experimental data taken in conditions of pure turbulence indicate the functional form (5) which we assume to be correct in what follows.

Despite its incompleteness, the theory of MDR is a brilliant intuition which is extremely important simplifying the following vectorial equations describing the convective flow in the cylinder, when applied to the nucleus:

$$\rho \frac{\partial(c_v T)}{\partial t} + \rho \mathbf{u} \cdot \nabla(c_v T) = \nabla \cdot (K \nabla T) - p \nabla \cdot \mathbf{u} + \psi \quad (6)$$

(equation of heat diffusion)

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \rho \mathbf{g} + \nabla \cdot \boldsymbol{P} \quad (7)$$

(Navier–Stokes equation)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (8)$$

(equation of continuity)

where (with the sum rule convention):

$$\psi = \mu(2e_{ij}^2 - 2/3 e_{jj}^2)$$

$$e_{ij} = \text{strain tensor} = 1/2\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$\boldsymbol{P}$ = stress tensor with components:

$$P_{ij} = -p\delta_{ij} + 2\mu e_{ij} - 2/3 \mu \delta_{ij} e_{kk}$$

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j. \end{cases}$$

In the nucleus the vectorial equations (6)–(8) can be transformed into scalar form according, for instance, to a reference frame with the vertical axis directed from top to bottom of the cylinder and using the same conditions (1) and (2) of MDR.

We must distinguish two possibilities:

(i) $T_0 > T_s$ (cooling)

The velocity $\mathbf{u}$ is opposite to $\mathbf{g}$ and its modulus v is defined by

$$\mathbf{u} = -\mathbf{k}v.$$

(ii) $T_0 < T_s$ (warming)
The velocity $\mathbf{u}$ is

$$\mathbf{u} = \mathbf{k}v.$$

We notice that in both cases $\partial T/\partial z < 0$ because in the gravity field the warmer liquid must ascend.

This fact implies different signs in the scalar equations and in the conditions where z is included. In what follows we ascribe the upper sign to cooling and the lower to warming. The equations are:

$$\frac{\partial T}{\partial t} \mp v\frac{\partial T}{\partial z} = \chi\frac{\partial^2 T}{\partial z^2} \quad (9)$$

$$\mp\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g \quad (10)$$

$$v\frac{\partial \rho}{\partial z} = \pm\frac{\partial \rho}{\partial t} \quad (11)$$

which are to be solved with the conditions:

on T:

$$T(z,0) = T_s + (T_0 - T_s)\exp \pm(0.35 - 0.7z/h) \quad (12)$$

$$\lim_{t\to\infty} T(z,t) = T_s \quad (13)$$

on v:

$$\begin{cases} \lim_{t\to 0} v(t) = 0 & (14) \\ \lim_{t\to\infty} v(t) = 0 & (15) \end{cases}$$

on ρ:

$$\begin{cases} \rho(z,0) = \rho[T(z,0)] \\ \quad = \rho(T_s)[1 - \beta(T_0 - T_s) \\ \quad \times \exp \pm(0.35 - 0.7z/h)] & (16) \\ \lim_{t\to\infty} \rho(z,t) = \rho(T_s) & (17) \end{cases}$$

implying that at $t = 0$ $\rho(T)$ is of the form

$$\rho(T) = \rho(T_s)[1 - \beta(T - T_s)]$$

Equations (12) and (16) deserve attention because they are an important turning point of our model. As a matter of fact it is extremely difficult to treat the nucleus if we assume an infinite temperature gradient at $t = 0$ at the boundary. Our model assumes a very rapid formation of the 'couche limite' and the nucleus (according to MDR data, this happens in a matter of a few seconds) and an effective initial distribution of temperature which generates the corresponding density distribution. The constants are chosen in equations (12) and (16) to give the exact values of T_0 and $\rho(T_0)$ in the centre of the cylinder.

The preceding system of equations and conditions can be solved *only if* the term $\partial^2 T/\partial z^2$ is negligible compared to $\partial T/\partial t$ and to $v\partial T/\partial z$. This is true for velocity $v \gg 10^{-4}$ cm s^{-1}, i.e. *when convection is strongly dominant upon conduction.*

Under these circumstances we have only two equations and three unknowns (p, v and T). However, we can solve the system by observing that the MDR functions (3) and (4) are particular solutions and that the new solution must contain additional terms to account for the inflection point and the correct conditions (12)–(17). The general solution is thus obtained by separating the z and t variables and assuming that $T(t)$ is of the form

$$T = T_s + A\exp(\mp\eta z)\sum_n \alpha_n/(at + b)^n \quad (18)$$

where A, η, α_n and b are constants to be determined, while a is the already known constant by MDR. The value of b, according to MDR, is 1.

We notice that the coupled functions

$$v = \omega_n/(at + b)$$

and

$$T = T_s + A\exp(\mp\eta z)/(at + b)^n$$
$$(\omega_n = \text{constant})$$

for any n provide a solution of the equation

$$\frac{\partial T}{\partial t} \mp v\frac{\partial T}{\partial z} = 0. \quad (19)$$

The MDR solution corresponds to the particular case of $n = 3$.

We have now only to determine the constants A, η and α_n. Furthermore we must chose the relevant terms of the series in equation (18). This last task requires quite tedious mathematics based on the following assumptions:

(a) Equation (18) must be a perturbation of equation (3).
(b) The function must have only one inflection point and to be always decreasing for $t > 0$.
(c) The conditions (12)–(14) must be satisfied remembering that from equation (19) at $t = 0$, v must equal zero, i.e. $\partial T/\partial t|_{t=0}/\partial T/\partial z|_{t=0} = 0$.

This approach gives the following functions describing the unsteady convection in a vertical cylinder submitted to a sudden horizontal temperature change $T_s - T_0$ at $t = 0$

$$T = T_s + (T_0 - T_s)wf(t) \quad (20)$$

$$v = -ah\dot{f}(t)/[0.7f(t)] \quad (21)$$

$$\rho(z,t) = \rho(T_s)[1 - \beta(T_0 - T_s)wf(t)] \quad (22)$$

$$p(z,t) = \rho(T_s)[g \pm \dot{v}(t)] \times [z - \beta h(w - 1)f(t)/0.7] + \text{const.} \quad (23)$$

where $w = \exp[\pm(1.7 - 0.7z/h)]$

$$f(t) = [[-2\exp(\mp 1.35) - 1]/[2(at + 1)^4] + 1/(at + 1)^3 + [4\exp(\mp 1.35) - 1]/[2(at + 1)^2]]$$

$$a = K^*Gr^{1/3}Pr^{-2/3}v/[h(R - \delta)]. \quad (24)$$

Equation (20) fits well our data (see Figs. 2 and 3) with $K^* = 0.200 \pm 0.002$. These results are in principle applicable to any fluid and the value of K^* (which has been determined for water) should not depend upon the substance.

It is noticeable that *the density equation* (22) *is linear in T at any time and level* while the pressure equation (23), apart from terms which are very small for rather incompressible fluids like water, is $\rho(T_s)gz$ + constant, i.e. coincident with the well-known hydrostatic pressure when temperature and density are constant. This behaviour agrees well with a reasonable prediction about these quantities.

Let us now focus our attention on the inflection point of equation (20) and observe that it provides a criterion to determine the effectiveness of convection in the centre of a cylinder. In fact if in the experiments of Figs. 2 and 3 we compare the time t_f, corresponding to the inflection point of the experimental curve, with the value t_f', calculated at the inflection point of the conduction curve, and observe that these inflections occur at almost the same ordinate regardless of the experimental conditions chosen, we can write that (within the error limits):

if $t_f/t_f' < 1$ the curve is convective
if $t_f/t_f' = 1$ the curve is conductive.

To carry on this comparison on theoretical grounds we can write the universal equations of convection and conduction in the centre of the cylinder:

$$\text{convection } y = w_1[(-2w_1^{-1} - 1)/[2(K^*x + 1)^4] + (K^*x + 1)^{-3} + (4w_1^{-1} - 1)/[2(K^*x + 1)^2]] \quad (25)$$

where $w_1 = \exp(\pm 1.35)$

$$\text{conduction } y' = \sum_{m=0}^{m=\infty} [\exp(-z^2x')]/[z_m J_1(z_m)] \quad (26)$$

where

$$x = at/K^* \quad (27)$$

$$x' = \chi t'/(R - \delta)^2. \quad (28)$$

At the inflection points:

$$x_f/x_f' = a(R - \delta)^2 t_f/(K^*\chi t_f'). \quad (29)$$

Recalling equation (24) and substituting in equation (29) we obtain finally:

$$t_f/t_f' = (x_f/x_f')\{\nu\chi/[\beta g \Delta T(R - \delta)^3]\}^{1/3}.$$

Convection occurs if $t_f/t_f' < 1$, i.e.

$$\beta g \Delta T(R - \delta)^3/(\nu\chi) > (x_f/x_f')^3. \quad (30)$$

In equation (30) the first member is the well-known expression for *Ra*, the Rayleigh number for layers, where the thickness is replaced by the reduced radius of the nucleus $R - \delta$.

This dimensionless group must be larger than $(x_f/x_f')^3$ in order to have significant heat transfer due to convection in the centre of the cylinder submitted to the described conditions. This critical number R_c can be calculated from the universal curves (25) and (26). We find that $R_c = (x_f/x_f')^3 = 1600 \pm 50$. This last error is due to the approximation of 1% on K^*.

An estimation of R_c is also provided by the data shown in Fig. 4 where a critical diameter of $\simeq 1.7$ cm is associated with a $\Delta T = 1°C$. If δ is assumed to be 0.3 cm, the reduced radius $R - \delta$ is 0.55 cm and we obtain $R_c = 1250$ while a value of $R_c = 1600$ corresponds to a reduced radius of 0.60 cm.

4. CONCLUSIONS AND PERSPECTIVES

This work has several potential applications because it solves a problem of convection in the non-linear region and in the unsteady state. Our treatment may be considered as a method of bridging the gap between linear and non-linear systems on the basis of the general relationships like the Rayleigh number rule which appear to be quite promising.

Our equations can be applied only to the geometrical center of the cylinder because they are derived under particular initial conditions and in the approximation of a model (due to MDR) which divides the convective cylinder into a 'couche limite' and a 'noyau central'. However, they can be modified to account for data taken out of centre.

We are now working to get more experimental information on this system in order to confirm the general validity of the model and also to apply it to water in the vicinity of 4°C where it presents a density maximum.

In the following papers we shall also discuss other effects we observed in the cylinder when submitted to various sudden temperature changes.

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CONVECTION NATURELLE VARIABLE ET NON LINEAIRE DANS UN CYLINDRE VERTICAL SOUMIS A UN GRADIENT THERMIQUE HORIZONTAL: MESURES DANS L'EAU ENTRE 6 et 21 C, ET MODELE THEORIQUE DE CONVECTION

Résumé—Le comportement non linéaire et variable de l'eau dans un cylindre vertical de cuivre jaune soumis initialement à un gradient thermique est étudié en suivant la décroissance de température au centre du cylindre. Les résultats expérimentaux sont interprétés au moyen d'un modèle théorique qui, à partir des équations, fournit la température, la vitesse, la pression et la densité dans le noyau. Les nouvelles équations sont comparées avec celles de la conduction pour évaluer la contribution de la convection au transfert de chaleur. Nos données montrent que lorsqu'un groupe caractéristique sans dimension qui a la forme d'un nombre de Rayleigh atteint une valeur critique de 1600 ± 50, le transfert de chaleur peut être décrit par une équation de conduction pure.

NICHTLINEARE, INSTATIONÄRE FREIE KONVEKTION IN EINEM SENKRECHTEN ZYLINDER MIT WAAGERECHTEM TEMPERATURGRADIENTEN: MESSUNGEN IN WASSER ZWISCHEN 6 UND 21°C UND EIN THEORETISCHES KONVEKTIONSMODELL

Zusammenfassung—Das nichtlineare, instationäre Verhalten von Wasser in einem senkrechten Messingzylinder, der einem horizontalen Temperaturgradienten ausgesetzt ist, wird untersucht, indem die zeitliche Temperaturänderung auf der Zylinderachse gemessen wird. Die Versuchsergebnisse werden mit Hilfe eines theoretischen Modells interpretiert, welches die Ableitung von Gleichungen für Temperatur, Geschwindigkeit, Druck und Dichte im Kern gestattet. Die Ergebnisse aus den neuen Gleichungen wurden mit denen der reinen Wärmeleitung verglichen, um eine Aussage über den konvektiven Anteil machen zu können. Unsere Ergebnisse zeigen, daß der Wärmetransport allein mit Hilfe der Wärmeleitgleichung beschrieben werden kann, wenn eine charakteristische Kennzahl, welche die Form einer Rayleigh-Zahl hat, einen kritischen Wert von 1600 ± 50 erreicht.

НЕЛИНЕЙНАЯ НЕСТАЦИОНАРНАЯ СВОБОДНАЯ КОНВЕКЦИЯ В ВЕРТИКАЛЬНОМ ЦИЛИНДРЕ С ГОРИЗОНТАЛЬНЫМ ТЕПЛОВЫМ ГРАДИЕНТОМ: ИЗМЕРЕНИЯ ДЛЯ ВОДЫ В ДИАПАЗОНЕ ТЕМПЕРАТУР ОТ 6 ДО 21°C И ТЕОРЕТИЧЕСКАЯ МОДЕЛЬ КОНВЕКЦИИ

Аннотация—Исследуется нелинейное нестационарное течение воды в вертикальном цилиндре из желтой латуни при начальном горизонтальном тепловом градиенте с последующей модификацией температуры в центре цилиндра. Экспериментальные результаты обрабатываются по теоретической модели, позволяющей выводить уравнения для температуры, скорости, давления и плотности в ядре потока. Эти уравнения сравниваются с известными для теплопроводности с целью оценить вклад конвекции в теплоперенос. Полученные результаты показывают, что при достижении безразмерным критерием, который имеет форму числа Рэлея, критического значения 1600 ± 50, теплоперенос может описываться уравнением чистой теплопроводности.